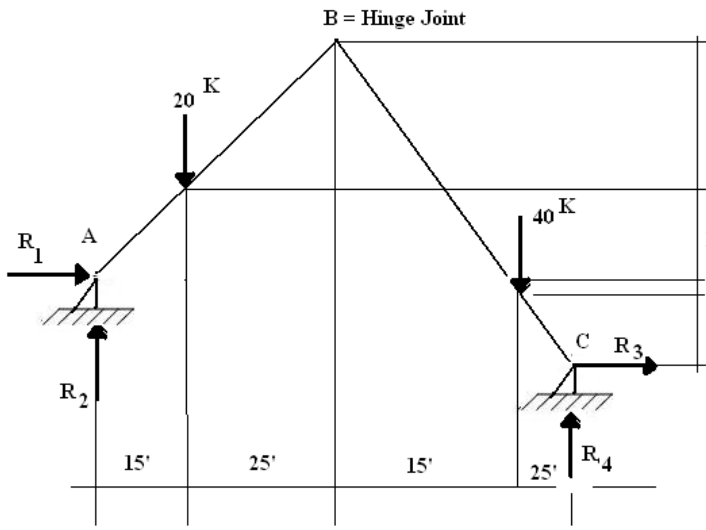
Review of Statics

Equation of Condition(s)

$(\Sigma M_{hingeB})_{BA} = 0 = (\Sigma M_{hingeB})_{BC}$



Method 1: solving 4 simultaneous equations

$\Sigma F_x = 0 \underset{=}{\rightarrow^+} R_1 + R_3 \equiv f_1(R_1, R_3) = 0$

$\Sigma F_y = 0 \underset{=}{\uparrow^+} R_2 + R_4 - 20^K - 40^K \equiv f_2(R_2, R_4) = 0$

$\Sigma M_A \underset{=}{\circlearrowleft^+} 0 = \underbrace{-(20^K)(15') - (40^K)(55') + (R_3)(12') + (R_4)(80')}_{f_3(R_3, R_4)} = 0$

$(\Sigma M_B)_{BA} \underset{=}{\circlearrowleft^+} 0 = (20^K)(25') + (R_1)(20') - (R_2)(40') = f_4(R_1, R_2) = 0$ ……(Eq.123)

Method 2:

$\Sigma M_A = 0 \ \circlearrowleft^+ = f_1(R_3, R_4)$

$(\Sigma M_B)_{BC} \underset{=}{\circlearrowleft^+} 0 = f_2(R_3, R_4)$

} Solve for R_3, R_4

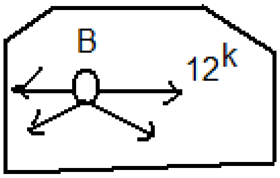
$\Sigma M_C = \circlearrowleft^+ \ 0 = f_3(R_1, R_2) = (40^K)(25') + (20^K)(65') - (R_1)(12') - (R_2)(80')$

$(\Sigma M_B)_{BA} = \circlearrowleft^+ \ 0 = f_4(R_1, R_2) = see\ Eq.(123)$

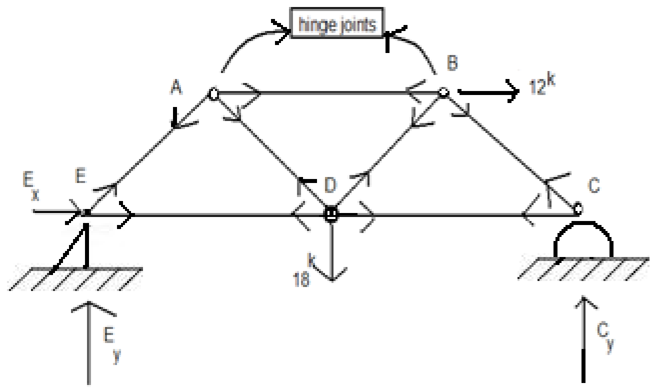
} Solve for R_1, R_2

Chapter 6 : Analysis of Structures

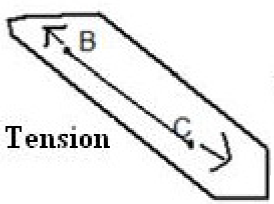
A) Method of Joints



FBD of Joint B

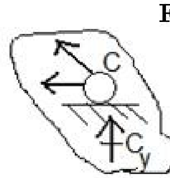


FBD of Member BC



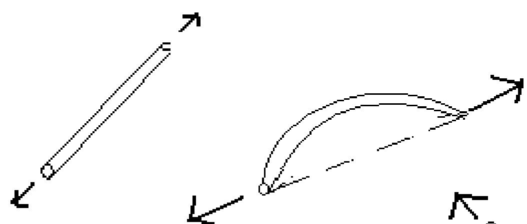
*Each TRUSS member
is connected by 2
"hinge joints"

FBD of Joint C



*A "hinge" join can
transfer forces, but NOT moments

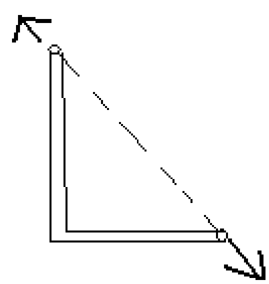
*Truss members can only carry
Axial forces



B) How to Compute Truss Member Forces

Step 1: Apply 3 equilibrium eqs.
To find 3 unknown reactions

$$\begin{cases} \Sigma F_x = 0 & \rightarrow + \\ \Sigma F_y = 0 & \uparrow + \\ \Sigma M_{z-axis} = 0 & \circlearrowleft + \end{cases}$$



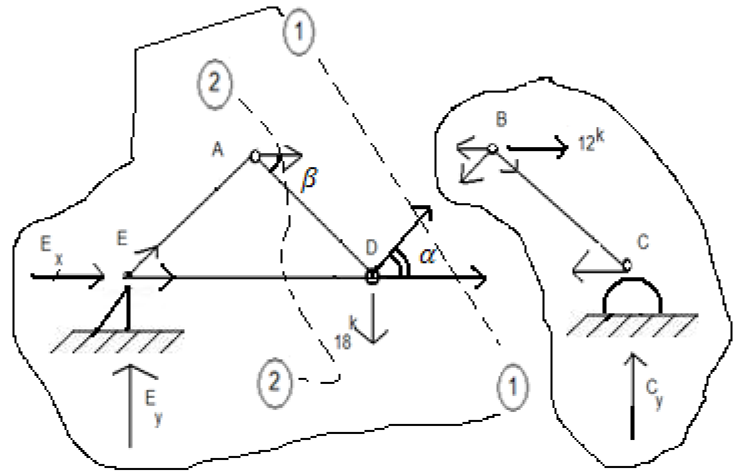
Step 2: At each joint which has 2 (or less) unknown member force(s),
apply 2 (force) equilibrium eqs
to find 2 (or less) member force(s)

$$\begin{cases} \Sigma F_x = 0 \\ \Sigma f_y = 0 \end{cases}$$

Step 3: Repeat step 2 until all (or few desired) member force(s) are found.

C) Method of Sections

This method is useful if one has interests to compute only few selected (not ALL) member forces, such as member forces AB, DB, AD



Step 1: Apply 3 equilibrium eqs. for the entire structure to find 3 unknown reactions.

Step 2: Make an "intelligent and honest" cut section (1)-(1)

- The cut should go through member(s) that you have interests
- The cut should NOT go through more than 3 members
- The cut must be completed, it should NOT end prematurely

Then, decide "which side" of the cut to draw FBD, say LEFT side!

Step 3: Apply 3 equilibrium eqs, using LEFT – FBD:

$$\sum M_D \overset{+\circlearrowleft}{=} 0 \Rightarrow f_1(F_{AB}, E_x, E_y) = 0 \rightarrow \boxed{solve\ for\ F_{AB}}$$

$$\sum F_x \overset{\rightarrow +}{=} 0 \Rightarrow f_2(E_x, F_{DC}, F_{DB}\cos\alpha, F_{AB}) = 0 \rightarrow \boxed{solve\ for\ F_{DC}}$$

$$\uparrow$$

$$\sum F_y \overset{\uparrow +}{=} 0 \Rightarrow f_3(E_y, -18^k, F_{DB}\sin\alpha) = 0 \rightarrow \boxed{solve\ for\ F_{DB}}$$

Notes: The remaining member force F_{AD} can be found by:

*Apply Method of Joint at joint A: $\Sigma F_x = 0$, $\Sigma f_y = 0$ } Solve for F_{AD}, F_{AE}

*or make another cut section (2)-(2), use LEFT-FBD, and

$\Sigma M_{@A} = 0 \Rightarrow f_4(F_{AB}, F_{AD}, F_{ED}, E_x, E_y) = 0 \Rightarrow$ Solve for F_{ED}

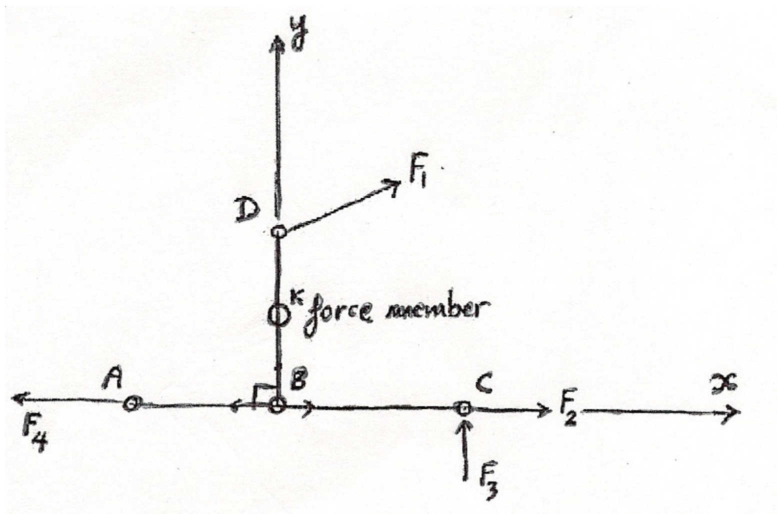
$\Sigma F_x = 0 \Rightarrow f_5(E_x, F_{ED}, F_{AD}\cos\beta, F_{AB}) = 0$

$\Sigma F_y = 0 \Rightarrow f_6(E_y, F_{AD}\sin\beta) = 0$

} Solve for F_{AD}

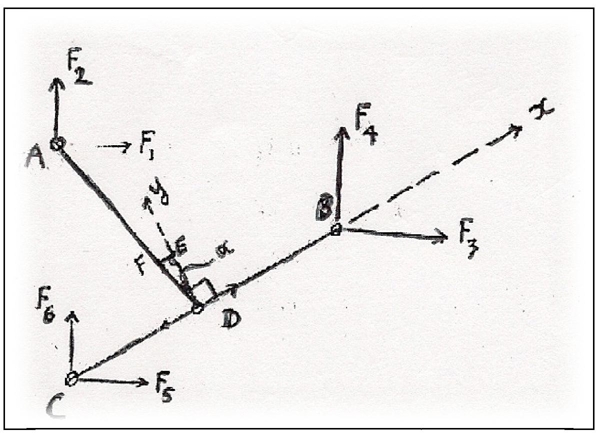
C_1) **Quick Ways to determine zero forces in truss structure**

At Joint B:
$\Sigma F_y \overset{\uparrow +}{=} 0 \;=>\; \boxed{0 = F_{BD}}$



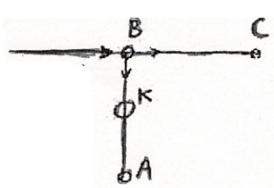
At Joint D:

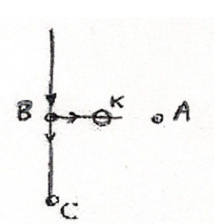
$\Sigma F_y \overset{\nwarrow +}{=} 0 = F_{DA}\cos\alpha => \boxed{F_{DA} = 0}$



Rules
"If we have 3 TRUSS members joining at a common joint (say, joint D), and if there are NO FORCES applied at the common Joint D, if two members form a 180° angle, such as members DB and DC, then the remaining member (such as member DA) should have Zero-force !"

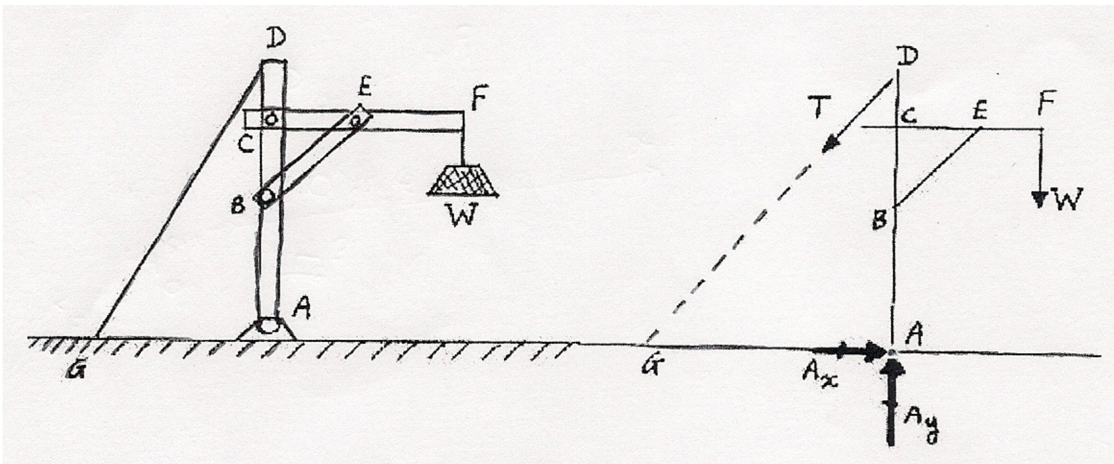
At Joint B;
$\Sigma F_y \overset{\uparrow +}{=} 0 = -F_{BA}$

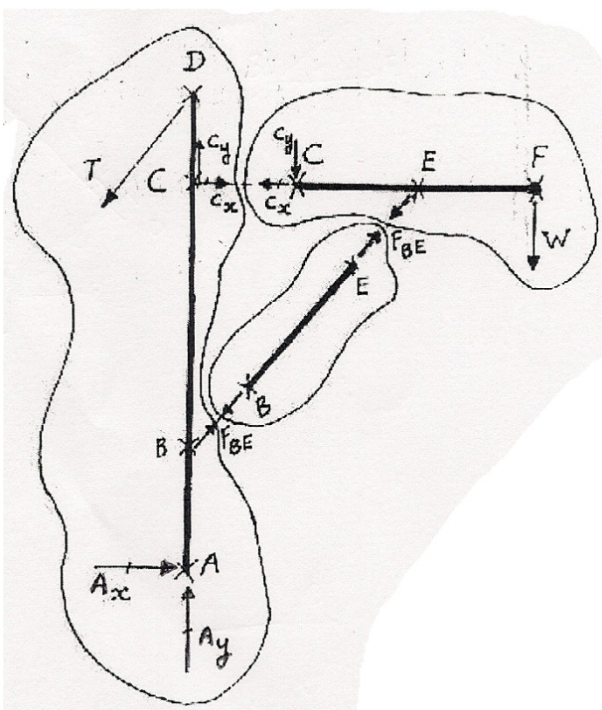




t Joint B:
$F_x \overset{\rightarrow +}{=} 0 = F_{BA}$

C_2) **Multi-Force (Beam, Frame) Members**





Notes:

a) Members AD and CF are multi-force members
b) Member BE is a 2 force (axial) member

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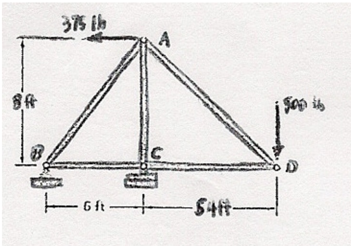
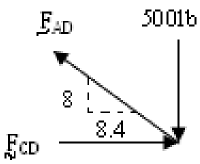


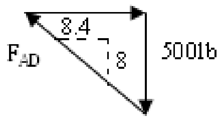
Problem 6.4
Using the method of Joints, determine the force in each member of the truss shown. State whether each member is in tension or compression

Solution:

Joint FBDs

- Joint D:



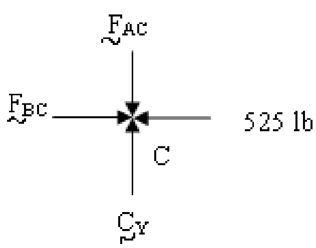


$$\frac{F_{CD}}{8.4} = \frac{F_{AD}}{11.6} = \frac{500 \text{ lb}}{8}$$

$F_{AD} = 725$ lb T
$F_{CD} = 525$ lb C

- Joint C:

$$=> \Sigma F_x = 0: \; F_{BC} - 525\, lb = 0$$



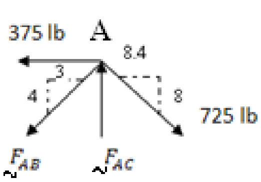
$F_{BC} = 525$ lb C

This is apparent by inspection, as is $F_{AC} = C_y$

$$=> \Sigma F_x = 0: \; \frac{8.4}{11.6}(725 \text{ lb}) - \frac{3}{5}F_{AB} - 375 \text{ lb} = 0$$

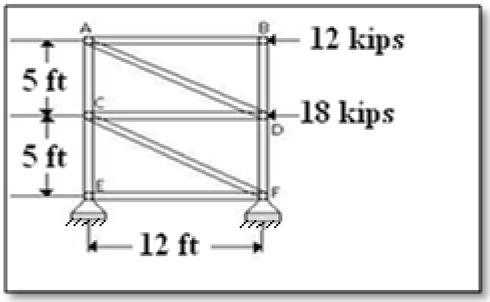
$F_{AB} = 250$ lb T

- Joint A :



$$\uparrow \Sigma F_y = 0 \; : F_{AC} - \frac{4}{5}(250 \text{ lb}) - \frac{8}{11.6}(725 \text{ lb}) = 0$$

$F_{AC} = 700$ lb C

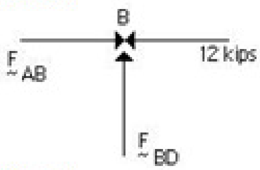


Problem 6.8
Using the method of joints, determine the force in each member of the truss shown.   State whether each member is in tension or compression.

Solution

Joint FBDs:

Joint B:

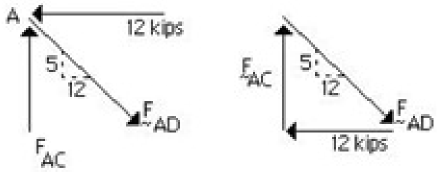


By inspection:

F_{AB}=12.00 kips C ◄

F_{BD}=0 ◄

Joint A:

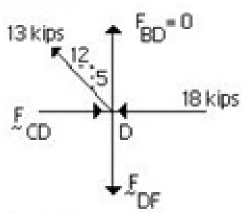


$$\frac{F_{AC}}{5} = \frac{F_{AD}}{13} = \frac{12\text{ kips}}{12}$$

F_{AC}=5.00 kips C ◄

F_{AD}=13.00 kips T ◄

Joint D:



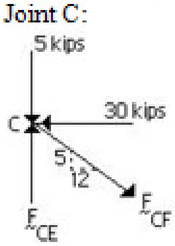
$\rightarrow\sum F_x = 0$: F_{CD} – 12/13 (13 kips) – 18 kips = 0

F_{CD}=30.0 kips C ◄

$\uparrow\sum F_y = 0$: 5/13 (13 kips) – F_{DF} = 0

F_{DF} = 5.00 kips T ◄

Joint C:



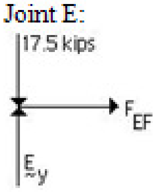
$\rightarrow\sum F_x = 0$: 30 kips – 12/13 F_{CF} = 0

F_{CF}=32.5 kips T ◄

$\uparrow\sum F_y = 0$: F_{CE} – 5 kips – 5/13 (32.5 kips)

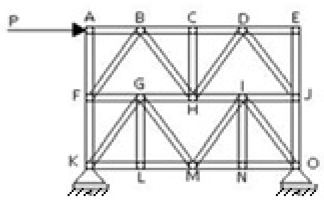
F_{CE}=17.50 kips C ◄

Joint E:



By inspection:

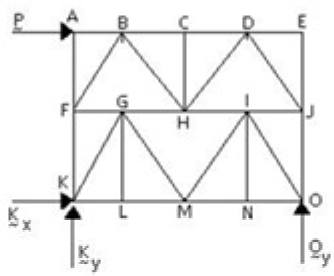
F_{CF}=0 ◄



Problem 6.34

For the given loading, <u>determine the zero-force members</u> in the truss shown.

Solution



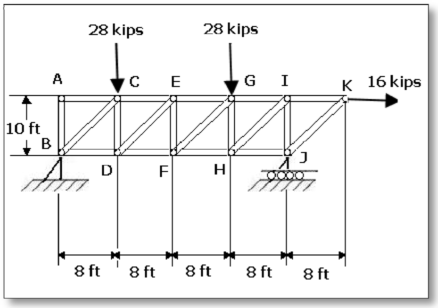
By inspection of joint A: $F_{AF} = 0$ ◀

By inspection of joint C: $F_{CH} = 0$ ◀

By inspection of joint E: $F_{DE} = F_{EI} = 0$ ◀

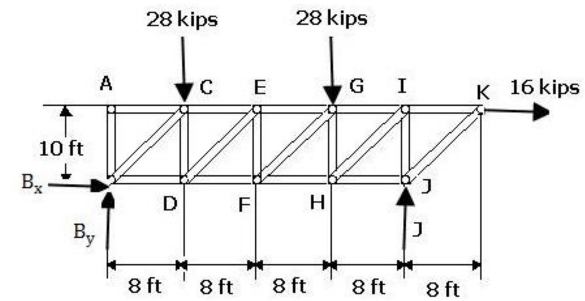
By inspection of joint L: $F_{GL} = 0$ ◀

By inspection of joint N: $F_{IN} = 0$ ◀



SAMPLE PROBLEM 6.2
Determine the force in members EF and GI of the truss shown.

SOLUTION:



Free-Body: Entire Truss. A free-body diagram of the entire truss is drawn; external forces acing on this free body consist of the applied loads and the reactions at B and J. We write the following equilibrium equations.

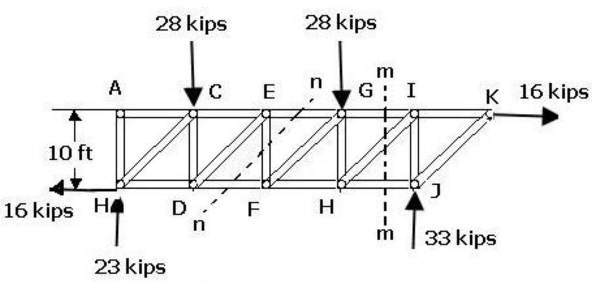
$$\circlearrowleft + \sum M_B = 0:$$

$$-(28\text{ kips})(8\text{ ft}) - (28\text{ kips})(24\text{ ft}) - (16\text{ kips})(10\text{ ft}) + J(32\text{ ft}) = 0$$

J = +33 kips $\quad J = 33\text{ kips} \uparrow$

$\overset{+}{\rightarrow} \sum F_x = 0:$ $\quad B_x$ + 16 kips = 0

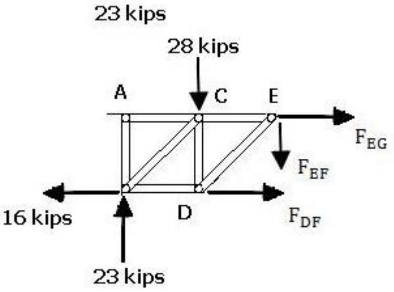
B_x = -16 kips $\quad B_x = 16\text{ kips} \leftarrow$



$$\circlearrowleft + \sum M_J = 0:$$

$$(28\text{ kips})(24\text{ ft}) + (28\text{ kips})(8\text{ ft}) - (16\text{ kips})(10\text{ ft}) - B_y(32\text{ ft}) = 0$$

$B_y = +23\text{ kips}$ $\quad B_y = 23\text{ kips} \uparrow$

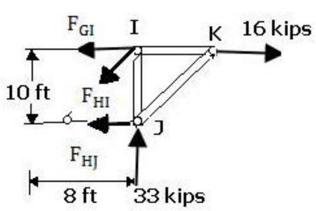


Force in Member EF. Section *nn* is passed through the truss so that it intersects member EF and only two additional members. After the intersected members have been removed, the left-hand portion of the truss is chosen as a free body. Three unknowns are involved: to eliminate the two horizontal forces, we write

$$+\uparrow \sum F_y = 0: \qquad +23\text{ kips} - 28\text{ kips} - F_{EF} = 0$$

$$F_{EF} = -5\text{ kips}$$

The sense of F_{EF} was chosen assuming member EF to be in tension; the negative sign obtained indicates that the member is in compression. $F_{EF} = 5$ kips C



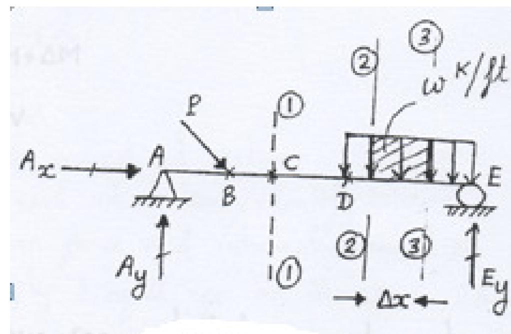
Force in Member GI. Section *mm* is passed through the truss so that it intersects GI and only two additional members. After the intersected members have been removed, we choose the right-hand portion of the truss as a free body. Three unknown forces are again involved; to eliminate the forces passing trough point H, we write

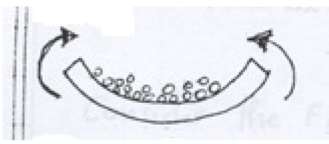
$$\circlearrowleft + \sum M_H = 0: \qquad (33\text{ kips})(8\text{ ft}) - (16\text{ kips})(10\text{ ft}) + F_{GI}(10\text{ ft}) = 0$$

$$F_{GI} = -10.4\text{ kips} \qquad F_{GI} = 10.4\text{ kips} \quad \text{C}$$

Chapter 7:
Forces in Beams and Cables

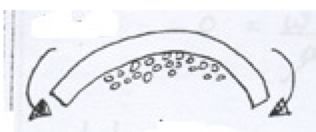
A. Sign Conventions :





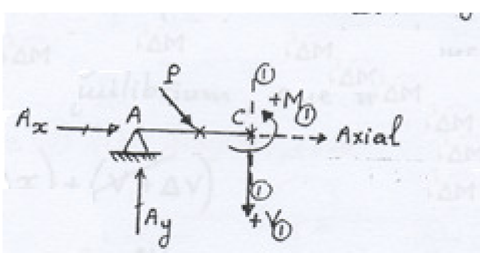
M=⊕

"Positive'' end-moments (because the "rice" still Remains inside the "bowl")



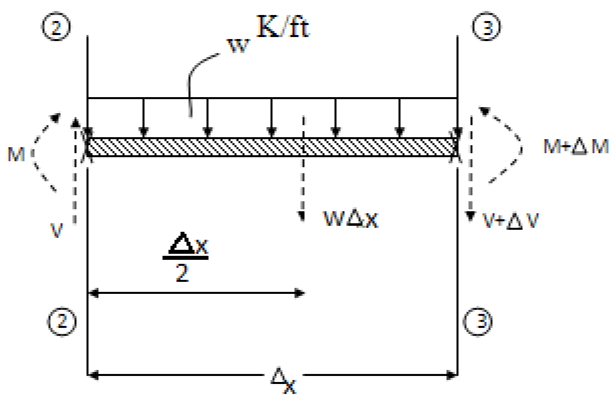
M= ⊖

"Negative" end-moments (because the "rice" will fall-off From the "bowl", hence no rice left to eat!)



FBD of beam segment AC, where V_1 & M_1 are internal shear force & bending moment in cut section (1) – (1)

B. Relations Between (Distributed) Loads "w", Interval Shear Force "v", and Bending Moment



Consider the FBD of beam segment between cut-sections (2) – (2) and (3) –(3).
For Equilibrium, one has:

$$\sum F_y \overset{\downarrow +}{=} 0 = -V + (w \cdot \Delta x) + (V + \Delta V)$$

$$0 = \frac{w\Delta x}{\Delta x} + \frac{\Delta V}{\Delta x} \Rightarrow \frac{\Delta V}{\Delta x} = -w$$

Let $\Delta x \to 0$ hence: $\boxed{\frac{dV}{dx} = -w}$

$$\sum M_{(2)} = 0 \overset{+\curvearrowleft}{=} -M - (w\Delta x)\left(\frac{\Delta x}{2}\right) + (M + \Delta M) - (V + \Delta V)\Delta x$$

$$0 = -w\left(\frac{\Delta x}{2}\right) + \frac{\Delta M}{\Delta x} - (V + \Delta V)$$

Let $\Delta x \to 0$; hence $\Delta V \to 0$; and above eq. becomes:

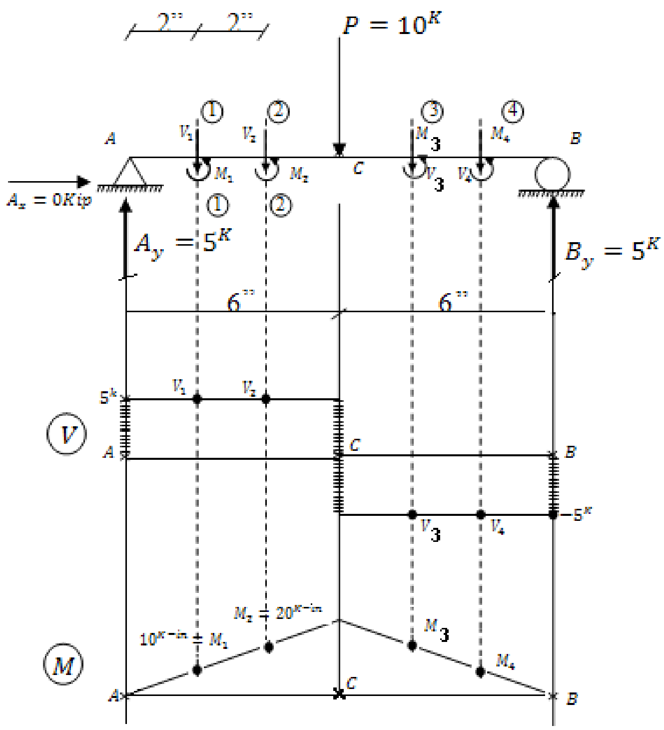
$$0 = \frac{+dM}{dx} - V; \text{ Or } \boxed{\frac{dM}{dx} = V}$$

Or $\int_{(2)}^{(3)} dM = \int_{(2)}^{(3)} dx$

$M_{(3)} - M_{(2)}$ =Area under shear curve, between cut-sections (2) and (3).

Basic Examples About Shear & Moment Diagrams

Example 1 (Concentrated Force Applied)



Notes:

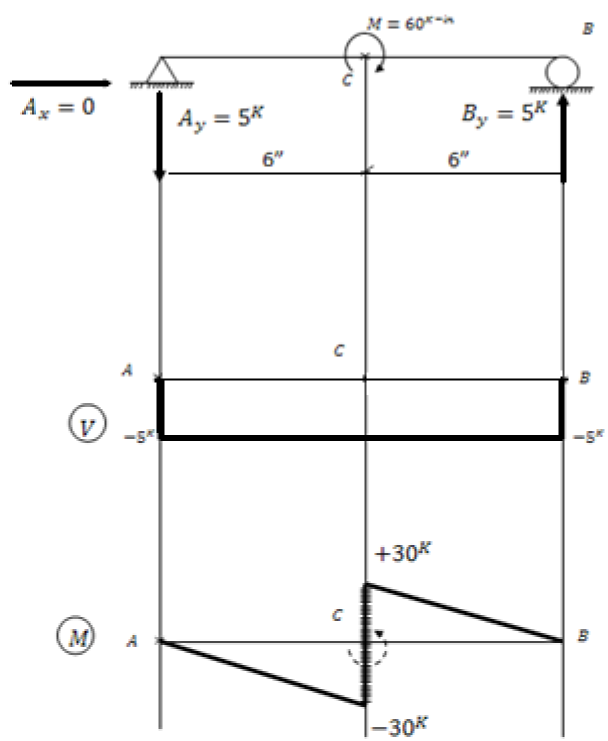
- Make several cut-sections
- At each cut-section, identify internal shear force V_i & internal moment M_i
- Apply 3 equil. eqs. On the "LEFT" FBD to solve for V_i & M_i
- Connect all the dots to create the (V) & (M) diagrams

**At concentrated force locations, there were discontinuous (V) values!

Example 2 (concentrated Moment Applied)

**At Concentrated moment locations,

There were discontinuous (M) values.



Quick Ways To Draw (V) & (M) Diagrams

To draw the internal shear diagrams:

"Look at the given beam structure, observe how the shear forces applied on the beam. Then, draw the (V) diagram accordingly"

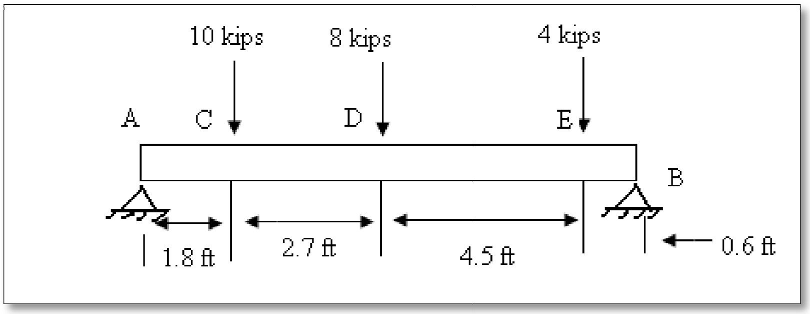
To draw the internal moment diagrams:

Step 1: Refer to the shear diagram, and compute the areas under the shear diagram sections

Step 2: Compute the initial value of internal moment (including the proper sign) say $M_{(1)}$

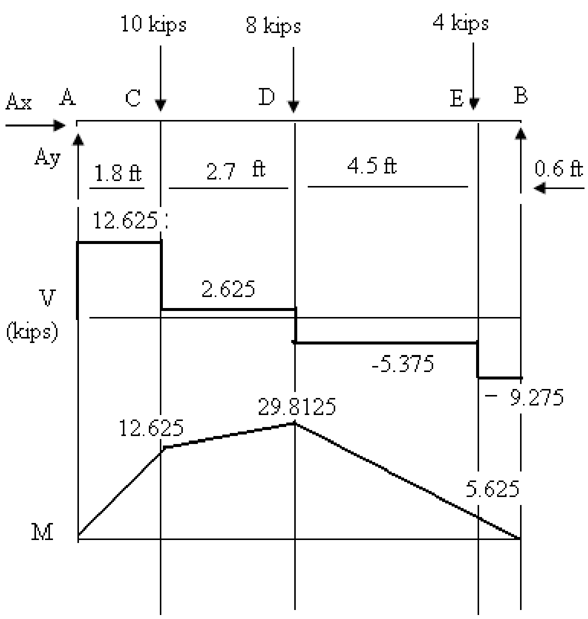
Step 3: Apply the formula: $M_{(2)} - M_{(1)}$ =area under (V) diagram,
Between sections (1) & (2) to compute $M_{(2)}$

Step 4: Decide how to connect the (two) dots in the (M) diagram
by referring to Eq. $\frac{dM}{dx} = V$; or $M_{(2)} - M_{(1)} = \int_{(1)}^{(2)} Vdx$



Problem 7.34
For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

Solution: (a)
FBD Beam:

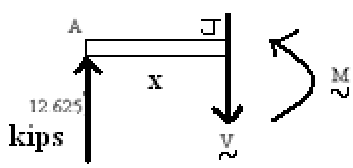


$\circlearrowleft \sum M_B = 0$:
(.6 ft)(4 kips) + (5.1 ft)(8 kips) + (7.8 ft)(10 kips) – (9.6 ft)
$A_y = 0$
$A_y = 12.625$ kips ↑

$\uparrow \Sigma F_y = 0 : 12.625 \text{ kips} - 10 \text{ kips} - 8 \text{ kips} - 4 \text{ kips} + B = 0$

B = 9.375 kips ↑

Along AC:

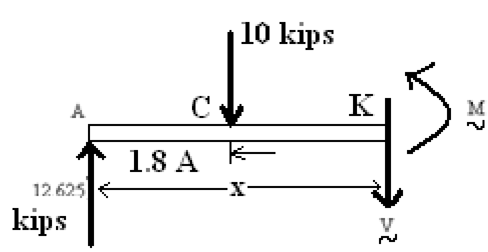


$\uparrow \Sigma F_y = 0 : 12.625 \text{ kips} - V = 0$

V= 12.625 kips
$\circlearrowleft \Sigma M_j = 0 : M - x(12.625 \text{ kips}) = 0$
M = (12.625 kips) x
M = 22.725 kip.ft at C

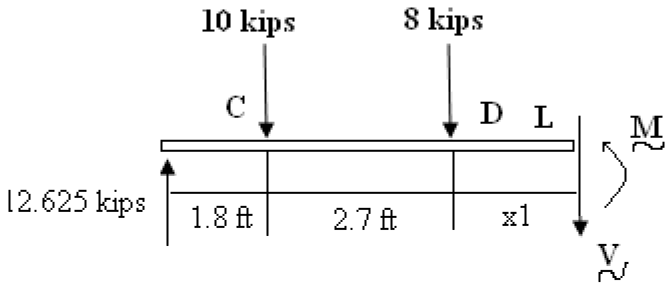
Along CD:

$\uparrow \Sigma F_y = 0 : 12.625 \text{ kips} - 10 \text{ kips} - V = 0$
V = 2.625 kips
$\circlearrowleft \Sigma M_k = 0$:
$M + (x - 1.8 \text{ ft})(10 \text{ kips}) - x(12.625 \text{ kips}) = 0$
M = 18 kip.ft + (2.625 kips)x
M = 29.8125 kip.ft at D (x=4.5 ft



Problem 7.34 Continued

Along DE:



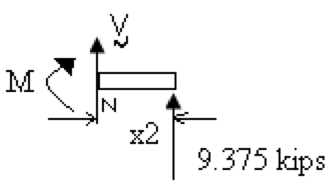
$\uparrow \Sigma F_y = 0$: (12.625 – 10 – 8) kips – V = 0, hence V = -5.375 kips

$\circlearrowleft\ \Sigma M_L = 0 : M + x_1(8\text{ kips}) + (2.7\text{ ft} + x_1)(10\text{ kips}) - (4.5\text{ ft} + x_1)(12.625\text{ kips}) = 0$

M = 29.8125 kip.ft - (5.375 kips) x_1

M = 5.625 kip.ft at E ($x_1 = 4.5$ ft)

Along EB:



$\uparrow \Sigma F_y = 0 : \; V + 9.375\text{ kips} = 0$ $\qquad$ $V = 9.375\text{ kips}$

$\circlearrowleft\ \Sigma M_N = 0 : \; x_2(9.375\text{ kip}) - M = 0$

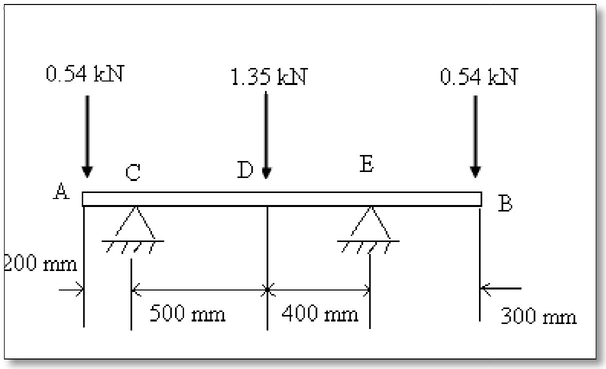
M = (9.375 kips) x_2

M = 5.625 kip.ft at E

(b) **From diagrams:**

$|V|_{max}$ = 12.63 kips on AC

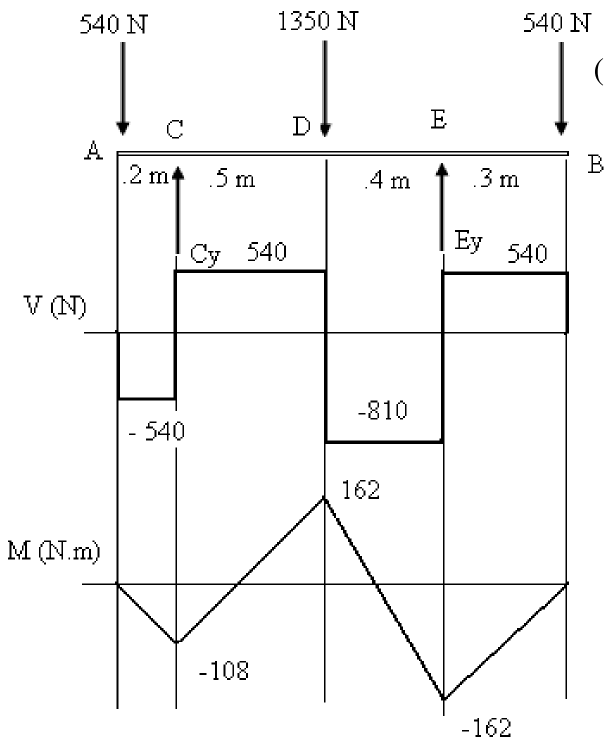
$|M|_{max}$ = 29.8 kip.ft at D



Problem 7.35

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

Solution:
(a)



FBD Beam: $\Sigma M_E = 0$:

$$(1.1\text{ m})(540\text{ N}) - (0.9\text{ m})C_y + (0.4\,m)(1350\,N) - (0.3\,m)(540\,N) = 0$$

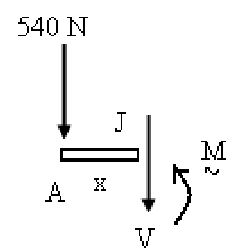
$$C_y = 1080\,N\ \uparrow$$

$\uparrow \Sigma F_y = 0$: $E_y = 1350\text{ N} \uparrow$

Along AC:

$\uparrow \Sigma F_y = 0$: -540 N- V= 0

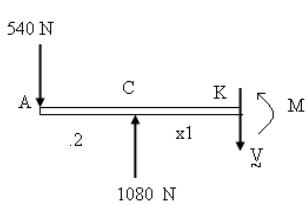
V = -540 N



$\circlearrowleft \Sigma M_j = 0$: $x(540\text{ N}) + M = 0$

M = -(540 N)x

Along CD:



$\uparrow \Sigma F_y = 0$: -540 N + 1080 N - V= 0 V = 540 N

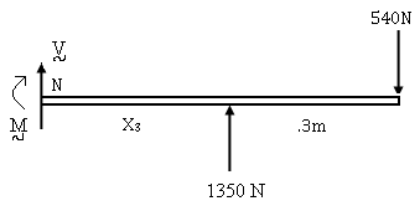
$\circlearrowleft \Sigma M_K = 0$: $M + (0.2\text{ m} + x_1)(540\text{ N}) - x_1(1080\text{ N}) = 0$

$M = -108\text{ N.m} + (540\text{ N})\,x_1$

$M = 162\text{ N.m at D } (x_1 = 0.5\text{ m})$

Problem 7.35 continued

Along DE:

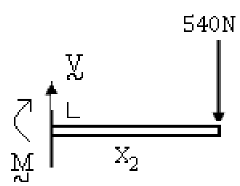


$\uparrow \Sigma F_y = 0$: V+ 1350 N – 540 N = 0 V = -810 N

$\Sigma M_N = 0$: $M + (x_3 + 0.3\ m)(540\ N) - x_3(1350\ N) = 0$

$M = -162\ N.m + (810\ N)x_3$
$M = 162\ N.m$ at D ($x_3 = 0.4$)

Along EB:



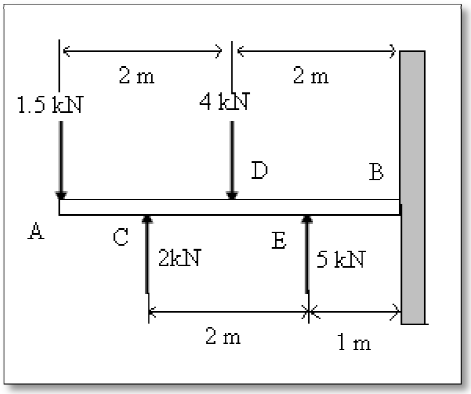
$\uparrow \Sigma F_y = 0$: V – 540 N = 0 V = 540 N

$\Sigma M_L = 0$: $M + x_2(540\ N) = 0$ M = -540 Nx_2
$M = -162\ N.m$ at E ($x_2 = 0.3\ m$)

(b) **From diagrams:**

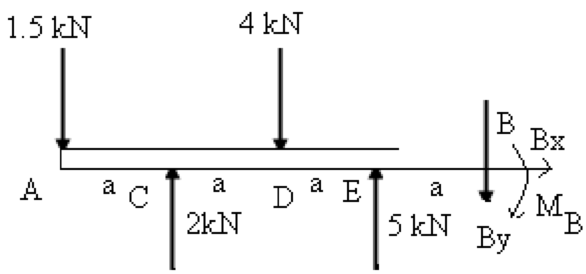
$|V|_{max}$ = 810 N on DE

$|M|_{max} = 162.0\ N.m$ at D and E



Problem 7.36

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.



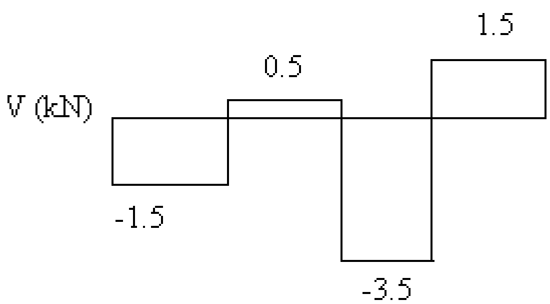
Solution:

(a) **FBD Beam**

a = 1 m $\Rightarrow$ $\Sigma F_x = 0$ $B_x = 0$

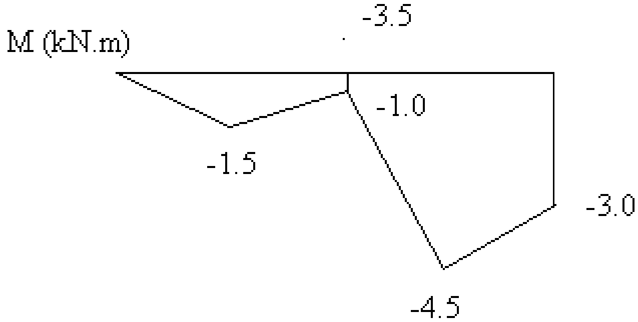
$\uparrow \Sigma F_y = 0$ $-1.5\ \text{kN} + 2\text{kN} - 4\ \text{kN} + 5\text{kN} - B_y = 0$

$B_y = 1.5\ \text{kN}\downarrow$

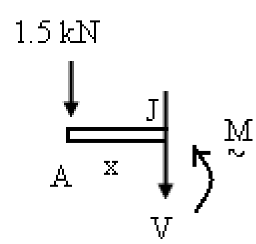


$\circlearrowleft \Sigma M_B = 0:$
$a[4(1.5\ \text{kN}) - 3(2\ \text{kN}) + 2(4\text{kN}) - 1(5\text{kN}] - MB = 0$

$M_B = (3\text{kN})a = 3\text{kN}\cdot\text{m}$ ↻



Along AC:



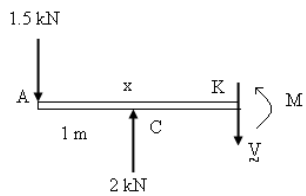
$\uparrow \Sigma F_y = 0$
$-1.5\text{kN} - V = 0$

V= -1.5kN

$\circlearrowleft \Sigma M_J = 0: M - x(1.5\ \text{kN}) = 0$ M(1 m)= - (1.5)x

Problem 7.36 Continued:

Along CD:

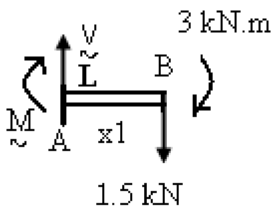


$\uparrow \Sigma F_y = 0 \;:\; -1.5\text{ kN} + 2\text{kN} - V = 0 \qquad V = 0.5\text{ kN}$

$\circlearrowleft\ \Sigma M_k = 0 \;:\; M + x(1.5\text{ kN}) - (x - 1\text{m})(2\text{kN}) = 0$

$M = -2\text{ kN·m} + (0.5\text{ kN})x \qquad M(2\text{m}) = -1\text{kN·m}$

Along EB:



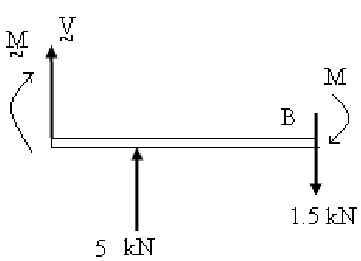
$\uparrow \Sigma F_y = 0 \qquad V - 1.5\ kN = 0 \qquad$ V= 1.5kN

$\circlearrowleft\ \Sigma M_L = 0: \qquad -M - x_1(1.5\text{kN}) - 3\text{kN·m} = 0$

$M = -3\text{ kN·m} - (1.5\text{ kN})\,x1$

$M(1\text{m}) = -4.5\text{ kN·m}$

Along DE:

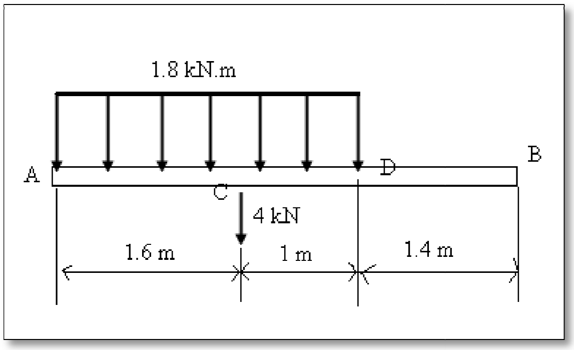


$\uparrow \Sigma F_y = 0 \;:\; V + 5\text{ kN} - 1.5\text{ kN} = 0 \qquad V = -3.5\text{ kN}$

Also M is linear here

(b) $|V|_{max} = 3.50$ kN along DE

$|M|_{max} = 4.50$ kN·m at E

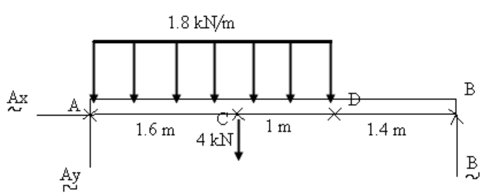


Problem 7.37

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

Solution:

(a) **FBD Beam**



$\circlearrowleft \Sigma M_A = 0$:

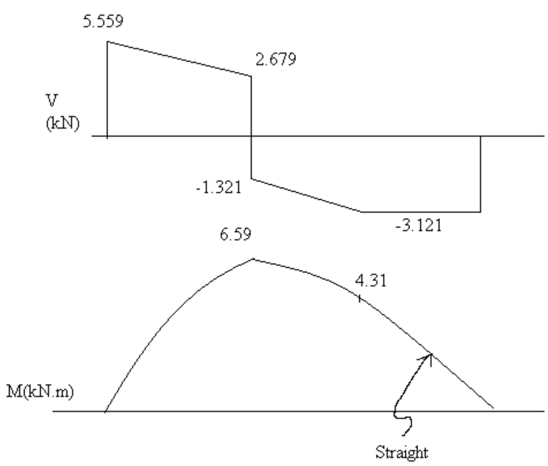
$-(1.3\text{ m})\left[\left(\frac{1.8\text{kN}}{\text{m}}\right)(2.6\text{ m})\right] - (1.6\text{ m})(4\text{ kN}) + (4\text{ m})\,B = 0$

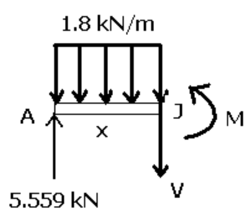
$B = 3.121\text{ kN} \uparrow$

$\uparrow \Sigma F_y = 0$:

$A_y - (1.8\text{ kN? m})(2.6\text{ m}) - 4\text{ kN} + 3.121\text{ kN} = 0$

$A_y = 5.559\text{ kN} \uparrow$





Along AC:

$\uparrow \Sigma F_y = 0$: $5.559\text{ kN} - (1.8\text{ kN/m})x - V = 0$

$V = 5.559\text{ kN} - (1.8\text{ kN/m})x$

$\circlearrowleft \Sigma M_J = 0$: $M + \frac{X}{2}[(1.8\text{ km})x] - x(5.559\text{ kN}) = 0$

$M = (5.559\text{ kN})x - (0.9\text{ kN/m})\,x^2$

Problem 7.37 Continued

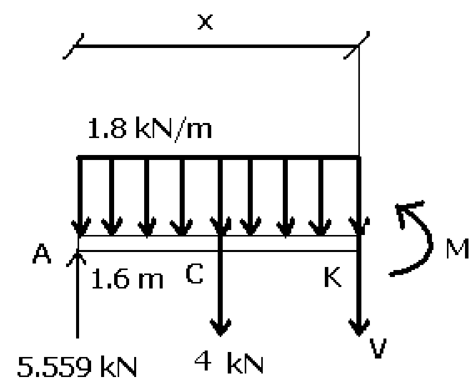
Along CD:

$\uparrow \Sigma F_y = 0$: $5.559\text{ kN} - x(1.8\text{ kN/m}) - 4\text{ kN} - V = 0$

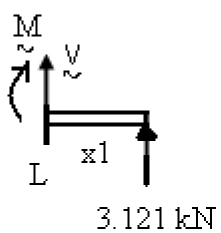
$$V = (1.559\text{ kN}) - (1.8\text{ kN/m})x$$

$\circlearrowleft$ $\Sigma M_K = 0$: $M + (x - 1.6\text{ m})(4\text{ kN}) + \frac{x}{2}[(1.8\text{ km})x] - x(5.559\text{ kN}) = 0$

$$M = 6.4\text{ kN.m} + (1.559\text{ kN})x - (0.9\frac{\text{kN}}{\text{m}})x^2$$



Along DB:



$\uparrow \Sigma F_y = 0$: $V + 3.121\text{ kN} = 0$

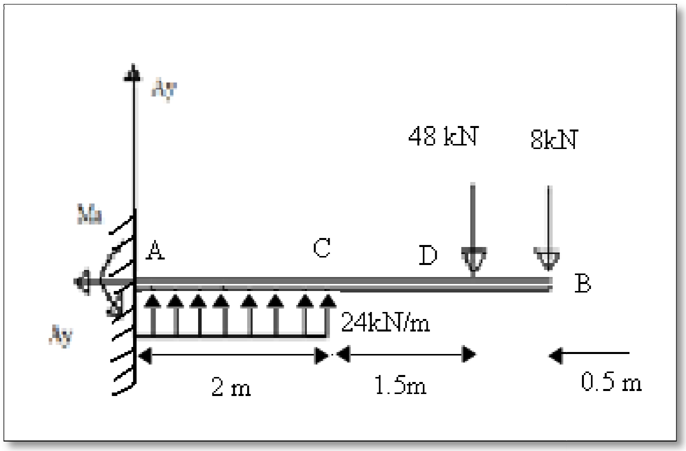
$$V = -3.121\text{ kN}$$

$\circlearrowleft$ $\Sigma M_L = 0$: $-M + x_1(3.121\text{ kN}) = 0$

$$M = (3.121\text{ kN})\,x_1$$

$|V|_{max} = 5.56\text{ kN at A}$

$|M|_{max} = 6.59\text{ kN.m at C}$

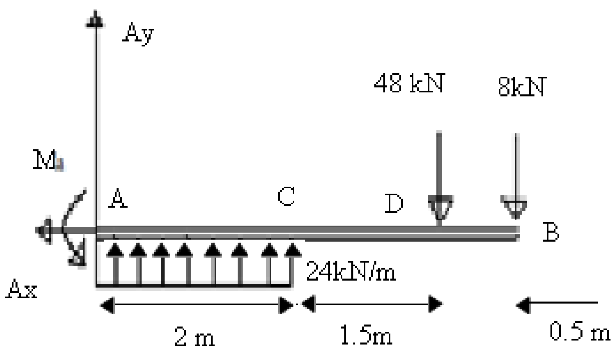


Problem 7.38

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

Solution:

(a) FBD Beam:



$\leftarrow \sum Fx = 0$: $\quad$ $\mathbf{A}x = 0$

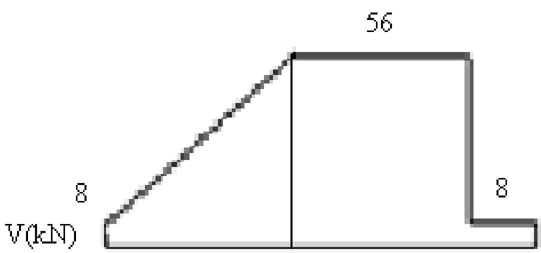
$\uparrow \Sigma F_y = 0 \quad \mathbf{A}y + (2m)(24kN/m) - 48kN - 8kN = 0$

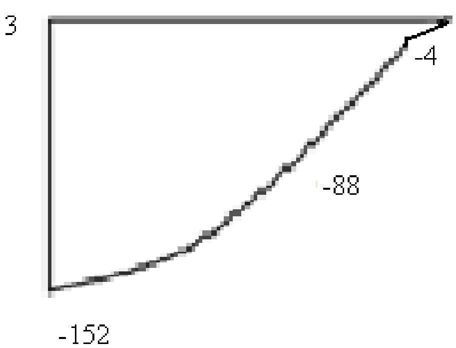
$\mathbf{A}y = 8kN \uparrow$

$\circlearrowleft \sum M_A = 0$:

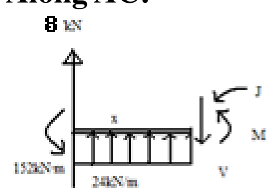
$M_A + (1m)(2m)(24kN/m) - (3.5m)(48kN) - (2m)(8kN) = 0$

$\mathbf{M}_A = 152 kN \cdot m$





Along AC:



$\uparrow \Sigma F_y = 0{:}\ 8kN + x(24\ kN.m) - V = 0$

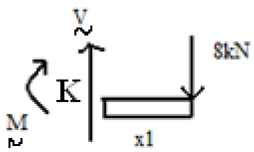
$V = 8\ kN + (24\ kN/m)x$

$\circlearrowleft \Sigma M_J = 0{:}\ M + 152\ kN.m - x(8\ kN) - \frac{x}{2}\left(24\frac{kN}{m}\right)x = 0$

$M = (12\ kN/m)\,x^2 + (8\ kN)x - 152\ kN.m$

Problem 7.38 Continued

Along DB:

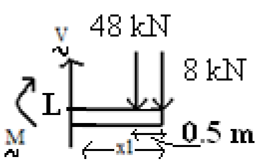


$\uparrow \Sigma F_y = 0$: $V - 8kN = 0$

$V = 8kN$

$\circlearrowleft \Sigma M_K = 0$: $M + x_1(8\ kN) = 0$ $M = -(8kN)\ x_1$

Along CD:



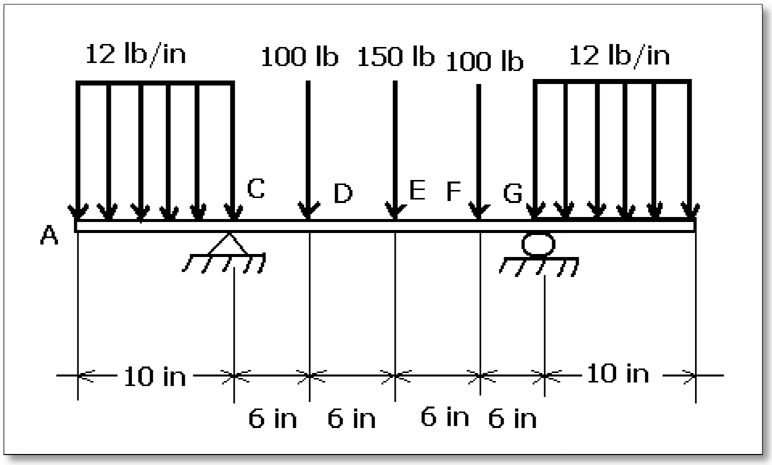
$\uparrow \Sigma F_y = 0$: $V - 48\ kN - 8\ kN = 0,$ $V = 56\ kN$

$\circlearrowleft \Sigma M_L = 0$: $M + (x_1 - 0.5\ m)(48\ kN) + x_1(8\ kN) = 0$ $M = 24\ kN.m - (56\ kN)x_1$

(b)

$|V|_{max} = 56.0$ kN along CD

$|M|_{max} = 152.0$ kN. m at A



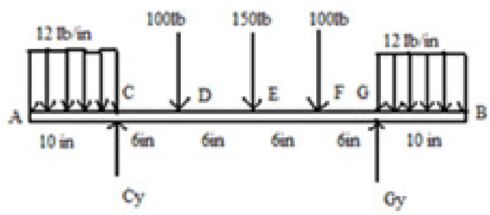
Problem 7.39

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

Solution:

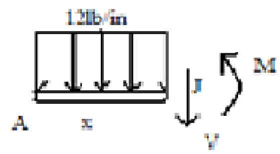
(a) By symmetry, $C_x = 0$, and

$C_y = G_y = \frac{1}{2}[2\left(12\frac{\text{lb}}{\text{in.}}\right)(10\text{ in}) + 2(100\text{ lb}) + (150\text{ lb})]$



$$C_y = G_y = 295\text{ lb}\uparrow$$

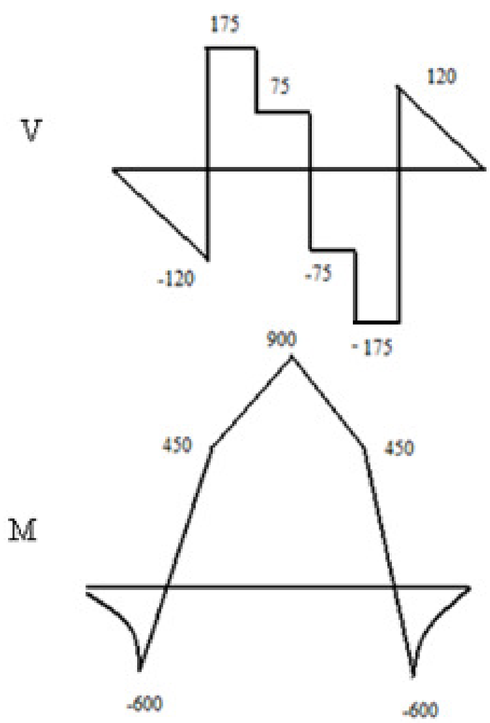
Along AC:



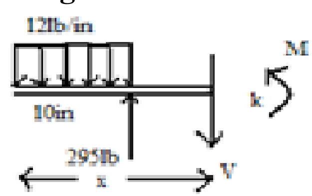
$\uparrow \Sigma F_y = 0{:}\ -\left(12\frac{\text{lb}}{\text{in.}}\right)x - V = 0$

$V = -\left(12\frac{\text{lb}}{\text{in.}}\right)x$

$\Sigma M_J = 0{:}\ M + \frac{x}{2}\left(12\frac{\text{lb}}{\text{in.}}\right)x = 0 \qquad M = -(6\frac{\text{lb}}{\text{in.}})x^2$



Along CD:



$\uparrow \Sigma F_y = 0{:}\ -\left(12\frac{\text{lb}}{\text{in.}}\right)(10\text{ in.}) + 295\text{ lb} - V = 0$

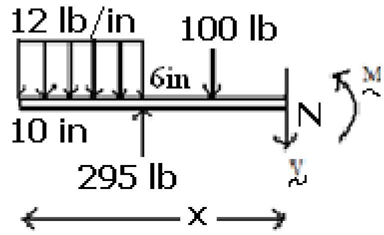
$V = 175\text{ lb}$

$\Sigma M_K = 0{:}\ M + (x - 5\text{ in.})\left(12\frac{\text{lb}}{\text{in}}.\right)(10\text{ in.}) - (x - 10\text{ in.})(295\text{ lb}) = 0$

$$M = -2350\text{ lb.in.} + (175\text{ lb})x$$

Problem 7.39 Continued

Along DE:



$$\uparrow \Sigma F_y = 0: -\left(12\frac{\text{lb}}{\text{in.}}\right)(10 \text{ in.}) + 295 \text{ lb} - 100 \text{ lb} - V = 0, \qquad V = 75 \text{ lb}$$

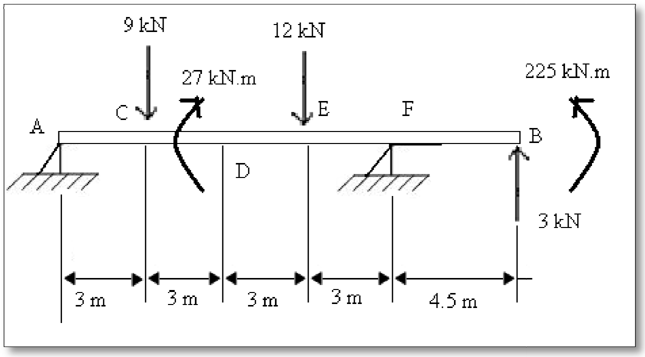
$$\circlearrowleft \Sigma M_N = 0: M + (x - 16 \text{ in.})(100 \text{ lb}) - (x - 10 \text{ in.})(295 \text{ lb}) + (x - 5 \text{ in.})\left(12\frac{\text{lb}}{\text{in.}}\right)(10 \text{ in.}) = 0$$

$$M = -750 \text{ lb.in} + (75 \text{ lb})x$$

(b) Complete diagrams using symmetry.

$|V|_{max} = 175.0$ lb along CD and FG

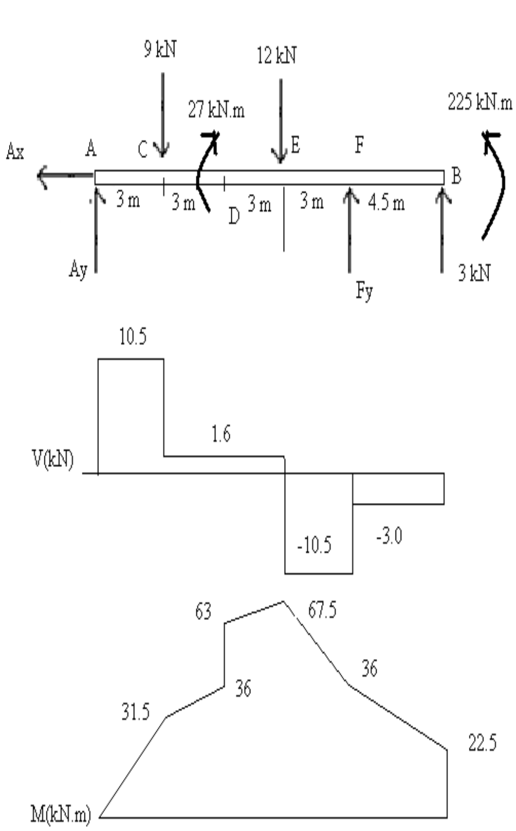
$|M|_{max} = 900$ lb. in at E



Problem 7.71

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

Solution:



(a) $\circlearrowleft \Sigma M_A = 0\!: -(3\text{m})(9\text{ kN}) - 27\text{ kN.m} - (9\text{m})(12\text{ kN}) + (12\text{m})F + (16.5\text{ m})(3\text{ kN}) + 22.5\text{ kN.m} = 0$

$$F = 7.5\text{ kN} \uparrow$$

$\Sigma F_y = 0\!: \quad A_y - 9\text{ kN} - 12\text{ kN} + 7.5\text{ kN} + 3\text{ kN} = 0$

$$A_y = 10.5\text{ kN} \uparrow$$

Shear Diag:

V is piecewise constant, with jumps at A, C, E, F, and B, equal to the forces there.

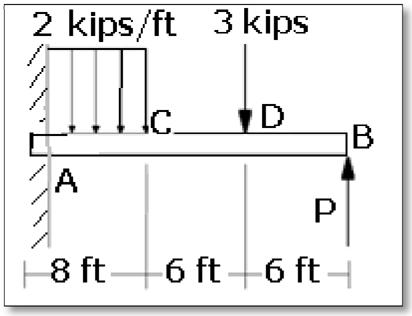
Moment Diag:

M is piecewise linear with jumps at D and B equal to the couples there.

$M_C = (10.5\text{ kN})(3\text{ m}) = 31.5\text{ kN.m}$
$M_{D^-} = 31.5\text{ kN.m} + (1.5\text{ kN})(3\text{ m}) = 36.0\text{ kN.m}$
$M_{D^+} = 36\text{ kN.m} + 27\text{ kN.m} = 63\text{ kN.m}$
$M_E = 63\text{ kN.m} + (1.5\text{ kN})(3\text{ m}) = 67.5\text{ kN.m}$
$M_F = 67.5\text{ kN.m} - (10.5\text{ kN})(3\text{m}) = 36\text{ kN.m}$
$M_{B^-} = 36.0\text{ kN.m} - (3\text{ kN})(4.5\text{ m}) = 22.5\text{ kN.m}$

(b) From the diagrams:

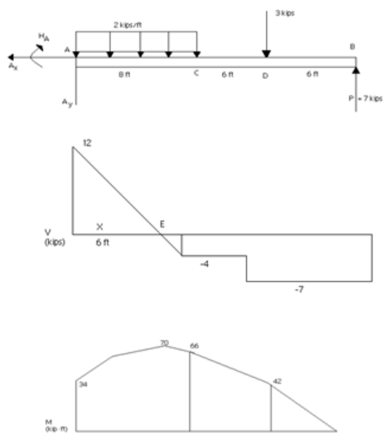
$$|V|_{max} = 10.50\text{ kN along AC and EF}$$
$$|M|_{max} = 67.5\text{ kN.m at E}$$



Problem 7.74
For the beam shown, draw the shear and bending-moment diagrams, and determine the maximum absolute value of the bending moment knowing that (a) P = 7 kips, (b) P = 10 kips.

Solution:

(a)



$\uparrow \Sigma F_y = 0$: $A_y (2\frac{\text{kips}}{\text{ft}})(8\text{ ft})$- 3kips + 7kips = 0

$$A_y = 12 \text{ kips} \uparrow$$

$$\circlearrowleft \Sigma M_A = 0: M_A + (4\text{ ft})\left(2\frac{\text{kips}}{\text{ft}}\right)(8\text{ ft}) - (14\text{ ft})(3\text{ kips}) - (20\text{ ft})(7\text{ kips}) = 0$$

$$M_A = 34 \text{ kip.ft} \circlearrowright$$

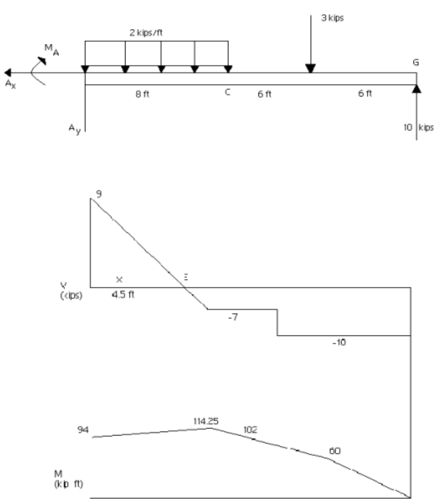
Shear Diag:
V jumps to 12 kips at A, then decreases at 2 kips/ft to -4 kips at C to D.
V drops 3 kips to -7 kips from D and B and jumps 7 kips to zero.

Note:
V = 0 where 12 kips – (2 kips/ft) x = 0, x = 6 ft.

Moment Diag:
M jumps to 34 kips.ft at A and then increases with decreasing slope to

34 kip·ft + $\frac{1}{2}$ (12 kips) (6 ft) = 70 kip·ft at E, and decreases by $\frac{1}{2}$ (4 kips) (2 ft) = 4 kip·ft, to 66 kip·ft at C. M then decrease by (4 kips)(6 ft) to 42 kip·ft at D, and by (7 kips) (6 ft) to zero at B.

$$|M|_{max} = 70 \text{ kip.ft at E}$$



(b)

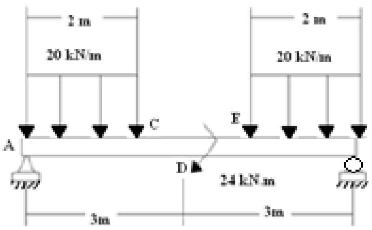
$$\uparrow \Sigma F_y \ 0: A_y - \left(2\frac{\text{kips}}{\text{ft}}\right)(8\text{ft}) - 3\text{ kips} + 10\text{ kips} = 0$$

$$A_y = 9 \text{ kips} \uparrow$$

$$\circlearrowleft \Sigma M_A = 0: M_A + (4\text{ ft})\left(2\frac{\text{kips}}{\text{ft}}\right)(8\text{ ft}) + (14\text{ ft})(3\text{ kips}) - (20\text{ ft})(10\text{ kips}) = 0$$

$$M_A = 94 \text{ kip.ft} \circlearrowright$$

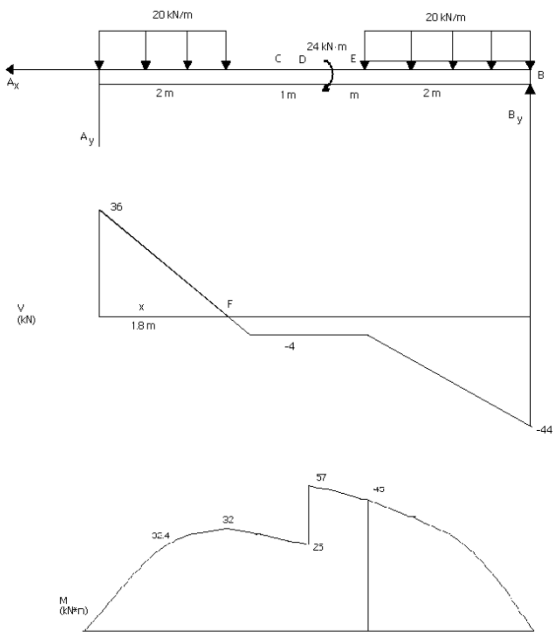
Shear Diag:
V jumps to 9 kips at A, then decreases, at 2 kips/ft, to -7 kips at C to D, drops 3 kips to -10 kips from D to B and jumps 10 kips to 0.
Note: V = 0 where 9 kips – (2 kips/ft) x = 0, x = 4.5 ft.



Problem 7.76

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the location and magnitude of the maximum absolute value of the bending moment.

Solution:



$$\overset{+}{\circlearrowleft}\ \Sigma M_A = 0:$$

$$(6\text{m})B_y - (1\text{ m})\left(20\frac{\text{kN}}{\text{m}}\right)(2\text{ m}) - 24\text{ kN.m} - (5\text{m})\left(20\frac{\text{kN}}{\text{m}}\right)(2\text{m}) = 0$$

$$B_y = 44\text{ kN}\uparrow$$

$$\uparrow \Sigma F_y = 0:\ A_y - 2\left(20\frac{\text{kM}}{\text{m}}\right)(2\text{ m}) + 44\text{ kN} = 0$$

$$A_y = 36\text{ kN}\uparrow$$

Shear Diag:

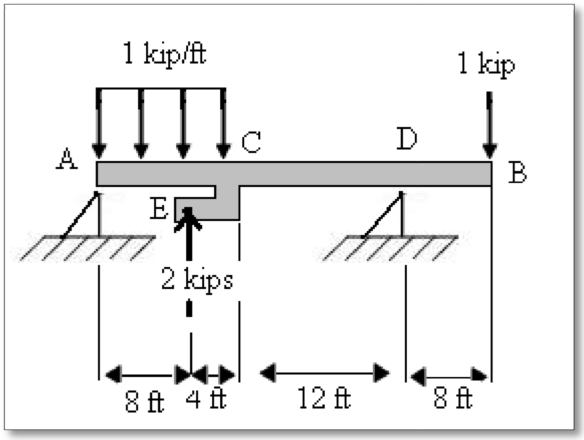
V jumps to 36 kN at *A*, then decreases with slope -20 kN/m to -4 kN at *C*, is constant to *E*, then decreases with slope -20 kN/m to -44 kN at *B*.

Note: $V = 0$ at *F* where 36 kN – (20 kN/m)$x = 0$, $x = 1.8$ m.

Moment Diag:

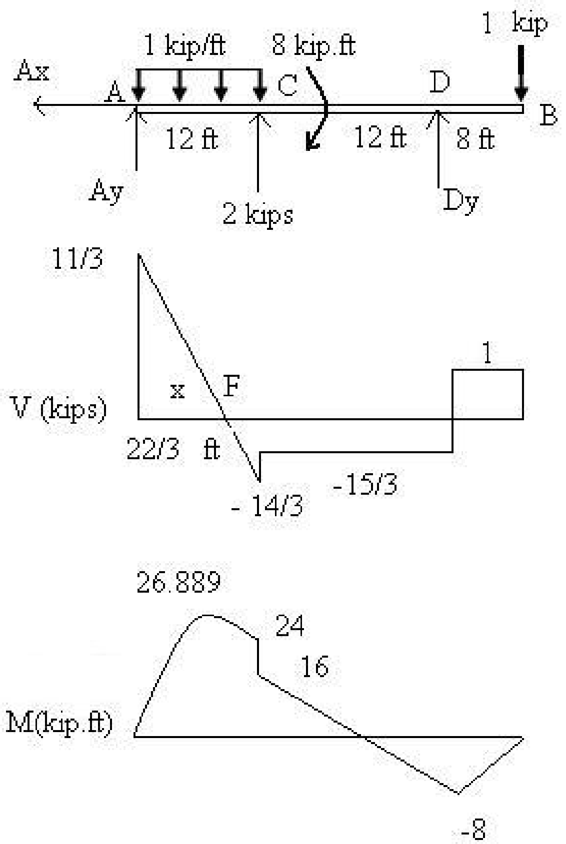
Starting at zero *M* increases with decreasing slope to $\frac{1}{2}$ (36 kN)(1.8 m) = 32.4 kN·m at *F*, decreases by $\frac{1}{2}$ (4 kN)(0.2 m) to 32 kN·m at *C*, then with slope -4 kN to 28 kN·m at *D*, where it jumps to 52 kN·m, *M* decreases with slope -4 kN to 48 kN·m at *E*, then with increasingly negative slope by $(\frac{4+44}{2}\text{ kN})$ (2 m) to zero at *B*.

$$|M|_{max} = 52\text{ kN}\cdot\text{m (at } D) \blacktriangleleft$$



Problem 7.79
Solve Prob. 7.78 assuming that the 2-kip force applied at E is directed upward.

Problem 7.78: For Beam AB, (a) draw the shear and bending-moment diagrams, (b) determine the location and magnitude of the maximum absolute value of the bending moment.



(a) **Note:** The 2 kip force at E has been replaced by the equivalent force and couple at C.

$$\circlearrowleft \sum M_{AB} = 0:\ -(6\ \text{ft})\left(1\frac{\text{kip}}{\text{ft}}\right)(12\ \text{ft}) + (12\ \text{ft})(2\ \text{kips}) - 8\ \text{kip.ft} + (24\ \text{ft})D_y - (32\ \text{ft})(1\ \text{kip}) = 0$$

$$D_y = \frac{11}{3}\ \text{kips}\ \uparrow$$

$$\uparrow \sum F_y = 0:\ A_y - \left(1\frac{\text{kips}}{\text{ft}}\right)(12\ \text{ft}) + 2\ \text{kips} - \frac{11}{3}\ \text{kips} - 1\ \text{kip} = 0$$

$$A_y = \frac{22}{3}\ \text{kips}\ \uparrow$$

Shear Diag:
Starting at $\frac{22}{3}$ kips at A, V decreases with slopes – 1 kip/ft to $\frac{-14}{3}$ kips at C, jumps 2 kips and remains constant at $\frac{-8}{3}$ kips to D, jumps $\frac{11}{3}$ kips and remains constant at 1 kip to B, drops to zero.
V = 0 at, where $\frac{22}{3}$ kip – (1 kip/ft)x = 0, x = $\frac{22}{3}$ ft

Moment Diag:
Starting from zero, M increases with decreasing slope to $\frac{1}{2}\left(\frac{22}{3}\ \text{kips}\right)\left(\frac{22}{3}\ \text{ft}\right)$ = 26.889 kip.ft at F.
M increases by $\frac{1}{2}\left(\frac{14}{3}\ \text{kips}\right)\left(\frac{14}{3}\ \text{ft}\right)$ to 16 kip.ft at C, jumps to 24 kip.ft, decreases with slope $\frac{-8}{3}$ kips to – 8 kip.ft at D, and finally increases with slope 1 kip to zero at B.

(b)

$$|M|_{max} = 26.9\ \text{kip.ft at F (7.33 ft from A)}$$